Written Exam at the Department of Economics summer 2017

Pricing Financial Assets

Final Exam

14 June 2017

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 2 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Consider an economy with securities available for investment at time $t = 0$ and with ultimate payment at time $t = T$. A stock and a zero coupon bond can be purchased at $t = 0$ at prices S_0 and e^{-rT}, respectively, where r is a constant. Neither of these securities has payments between time 0 and T.

   Assume that at time T the realization of two possible states of the world is revealed. The stock has a payoff of uS_0 and dS_0, depending on the state, where $u > \mathrm{e}^{rT} > d \geq 0$ are constants. The bond has a payment of 1 at $t = T$, irrespective of the state.

   (a) Explain what is meant by risk neutral probabilities and (Arrow-Debreu) state prices and determine these for this economy.

   (b) What is the forward price at $t = 0$ for delivery of the stock at $t = T$? What is the arbitrage argument?

   (c) Consider a forward contract on a stock maturing at time T. Compare the pricing of a European call option on the stock with the price of a European call on the forward contract on the stock, all maturing at T and having the same strike K.

2. Let $V(t)$ denote the probability of a borrower not defaulting before time $t \geq 0$, $V(0) = 1$.

   (a) We will often assume that $V(t)$ is weakly decreasing in t. Why?

   (b) Assume that V is differentiable. Define and interpret the continuously compounded hazard rate, $\lambda(t)$.

   (c) What is a ratings transitions matrix? What is the typical structure af such a matrix?

   (d) What assumptions are needed to use a ratings transition matrix to derive an estimate of the likelihood of a default of a rated issuer within a certain time horizon? And will this estimate be a real world or a risk neutral probability?

3. Let $R(t, T)$ denote the continuously compounded yield at time t of a zero coupon bond that matures at time $T > t$. Assume that R follows an Ito-process of the form

$$dR = \mu dt + \sigma dz$$

   where z is a Brownian motion, and μ and $\sigma > 0$ are bounded functions of R and t.

   (a) For a bond with given maturity T show that the volatility of the bond price $P(t, T)$ will converge to zero as t converges to T.

   (b) Now assume $\mu = m(k - R)$ and $\sigma = sR$, where m, k and s are positive constants. What is the process followed by the bond price?